Dependence of the Multiple-Device Oscillator Injection-Locking Range On the Number of Constituent Devices

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Abstract —An analysis of the behavior of an injection-locked multiple-device oscillator is presented. It is shown that the external Q of the oscillator circuit decreases towards a limiting value as the number of devices is increased. This limiting value is determined by the Q of an individual device. It is also shown that improvement of the locking range with an increase in the number of devices is rapid when the locking gain is very low and independent of the increase of the number of devices.

I. Introduction

Solid-state devices, such as Gunn's, IMPATT's, etc., are widely used for millimeter and microwave power generation. The limited power output of these devices, however, fails to meet the power requirements of many applications. This limitation is often overcome by the use of multiple-device oscillators [1], [2]. The injection-locking behavior of these oscillators has been the subject of study of many authors [3]–[6]. Experimentally, it is observed that with the increase in the number of devices the external Q of a multiple-device oscillator circuit decreases and its locking range increases [4]–[6].

Nogi *et al.* [6] have given a theoretical explanation for the decrease of external Q with the increase in the number of devices. This, however, does not provide a complete picture of the dependence of the locking range on the number of devices. In this paper, an attempt has been made to analytically relate the injection-locking range of a multiple-device oscillator with the number of its constituent devices. The behavior of external Q is also analyzed in the process.

II. Behavior of External Q

Fig. 1 shows the equivalent circuit of a multiple-device oscillator of the type analyzed by Kurokawa [3]. It consists of a number N of identical negative conductance devices, which are equally coupled (n:1) to a summing resonator. $y_d = g_d + jb_d$ is the admittance of an individual device. In the summing resonator circuit, $Y_L = G_L + jB_L$ is the load admittance and G_c is the loss conductance. C_c and L_c are the capacitance and inductance associated with the resonator. The constant current source i_0 represents the injection signal. It is assumed that the injection signal is too small to affect y_d appreciably and its frequency is very close to the free-running frequency (f_o) of the oscillator. From Fig. 1, the total capacitive susceptance seen by the injection signal is

$$B_t = 2\pi f_o\left(C_c + n^2 N C_D\right) \tag{1}$$

where C_D is the capacitance of an individual device. Since the injection signal is assumed to be very small using (1), the external Q may be approximated by

$$Q_{\text{ext}} = \frac{2\pi f_o}{G_L}\left(C_c + n^2 N C_D\right). \tag{2}$$

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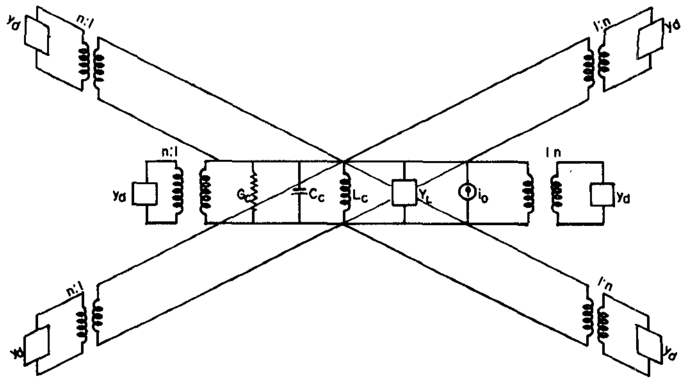

Fig. 1. Equivalent circuit of the multiple-device oscillator under consideration. Dots indicate the presence of other branches.

If $-g_o$ is the optimum negative conductance of an individual device and the oscillator circuit is optimized for maximum power generation, then

$$G_L = n^2 N g_o. \tag{3}$$

Substituting (3) in (2)

$$Q_{\text{ext}} = Q_D\left(1 + \frac{C_c}{n^2 N C_D}\right) \tag{4}$$

where

$$Q_D = \frac{2\pi f_o C_D}{g_o} \tag{5}$$

is the quality factor of an individual device. From (4), it is evident that Q_{ext} of a multiple-device oscillator decreases towards the limiting value of Q_D as the number of devices is increased.

Fig. 2 illustrates the variation of Q_{ext} with the number of devices for $Q_D = 6.28$, $C_c = 10$ pF, $C_D = 0.3$ pF, and $n = 0.4$. Recently Nogi *et al.* [6] reported similar behavior of Q_{ext} of a multiple-device oscillator.

III. Locking Range as a Function of the Number of Devices

From Kurokawa's analysis of multiple-device oscillators [3], it can be shown that the locking gain of the oscillator shown in Fig. 1 is

$$A = \frac{N g_o v_o^2}{2P_t}\left(1 + \frac{G_c}{n^2 N g_o}\right)^{-1} \tag{6}$$

where v_o and P_t are the RF voltage at the terminals of an individual device and injection power, respectively. The locking range of a multiple-device oscillator may be expressed by [3]

$$\Delta f_r = \frac{2 f_o F_D}{Q_{\text{ext}}\sqrt{A}} \tag{7}$$

where F_D is a device parameter.

From (4)–(7), it can be shown that

$$\Delta f_r = \frac{F_D}{\pi v_o C_D}\left(1 + \frac{C_c}{n^2 N C_D}\right)^{-1}\left[\frac{2P_t g_o}{N}\left(1 + \frac{G_c}{n^2 N g_o}\right)\right]^{1/2}. \tag{8}$$

This expression for the locking range is applicable when Q_{ext} and the locking gain vary simultaneously as the number of

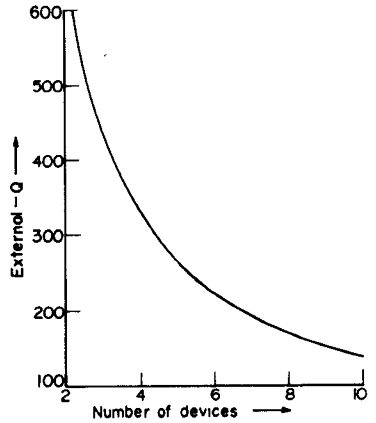

Fig. 2. External-Q as a function of the number of devices.

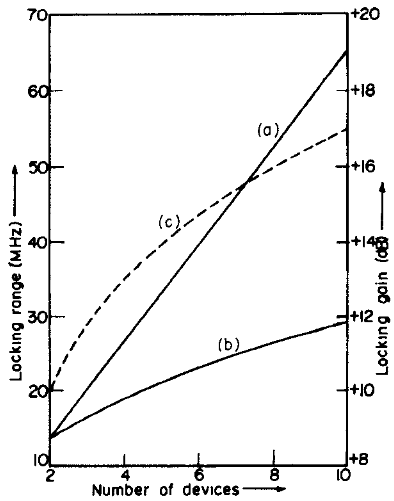

Fig. 3. Functions of the number of devices. (a) Locking range with constant gain. (b) Locking range with constant injection power. (c) Locking gain with constant injection power.

devices is changed. From (4), (5), and (7), it can be easily seen that

$$\Delta f_r = \frac{F_D g_o}{\pi C_D \sqrt{A}} \left(1 + \frac{C_c}{n^2 N C_D}\right)^{-1} \tag{9}$$

when the locking gain is kept constant by proportionately increasing P_i as the devices are increased in number.

Fig. 3 shows the variation of the locking range with an increase in the number of devices for a multiple-device oscillator of $F_D = 1.41$, $v_o = 18.26$ V, $C_D = 0.3$ pF, $C_c = 10$ pF, $n = 0.4$, $g_o = 3$m mho, and $G_c = 0.01$m mho. Two cases are shown. One obtained from (9) for a constant locking gain of 9.96 dB, which is the locking gain of the oscillator for $N = 2$ and $P_i = 100$ mW. For the other case, which is a plot of (8), injection power is constant at 100 mW and the locking gain increases with an increase in the number of devices (broken line in Fig. 3) along with a decrease of Q_{ext} (Fig. 2). The two cases thus illustrated show that, in general, the locking range of a multiple-device oscillator increases as its constituent devices are increased in number. A comparison of the two cases indicates that, with the gain constant at a low level, a larger deviation in the locking range is obtained as the devices are increased in number. In this case, full advantage of the fall in Q_{ext} with an increase in the number of devices is taken. When injection power is constant, as the devices are increased in number, the favorable effect of the corresponding decrease in Q_{ext} is diminished by the accompanying increase in the locking gain. As a consequence, the deviation in the locking range with an increase in the number of devices is not as large as it is in the constant gain case. Thus, when the constituent devices of a multiple-device oscillator are increased in number under a constant locking-gain condition, the resulting rise in the locking range is determined by an increase in the function Q_{ext}^{-1}. When the number of devices is increased, with the primary objective of increasing the locking gain, the dependence of the locking range on the number of devices is determined by the function $(Q_{\text{ext}} A^{\frac{1}{2}})^{-1}$.

IV. Conclusion

The analysis just presented shows that the Q_{ext} of a multiple-device oscillator falls towards a limiting value as the number of devices is increased. This limiting value happens to be the quality factor of an individual device. The decrease of Q_{ext} with an increase in the number of devices can be fully utilized for increasing the injection-locking range if the locking gain is kept constant at the minimum possible level. Such a measure, of course, will lead to an increasing demand on the injection power as the devices are increased in number. In many applications, the devices are increased in number to meet high-gain requirements. In such cases, the improvement of the locking range with an increase in the number of devices is comparatively less than what may be achieved with the locking gain remaining constant at the minimum possible level.

References

[1] K. J. Russel, "Microwave power combining techniques," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 472–478, May 1979.
[2] K. Chang and C. Sun, "Millimeter-wave power combining techniques," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 91–107, Feb. 1983.
[3] K. Kurokawa, "The single-cavity multiple-device oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 793–801, Oct. 1971.
[4] R. Aston, "Techniques for increasing the bandwidth of a TM_{010} mode power combiner," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 479–482, May 1979.
[5] Y.-E.Ma and Cheng-Sun, "1-W millimeter-wave Gunn diode combiner," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 1460–1463, Dec. 1980.
[6] S. Nogi and K. Fukui, "Locking behavior of a microwave multiple-device ladder oscillator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 253–262, Mar. 1985.

Invariant Definitions of the Unloaded Q Factor

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Abstract — **Equations are presented for computing the unloaded Q factor of a microwave resonator embedded in an impedance-transforming lossless, reciprocal two-port. Knowledge of the transformation properties of the two-port is not required.**

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